National Numeracy Network Conference
Michigan State University
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MATHEMATICS AND SOCIAL JUSTICE MODULES

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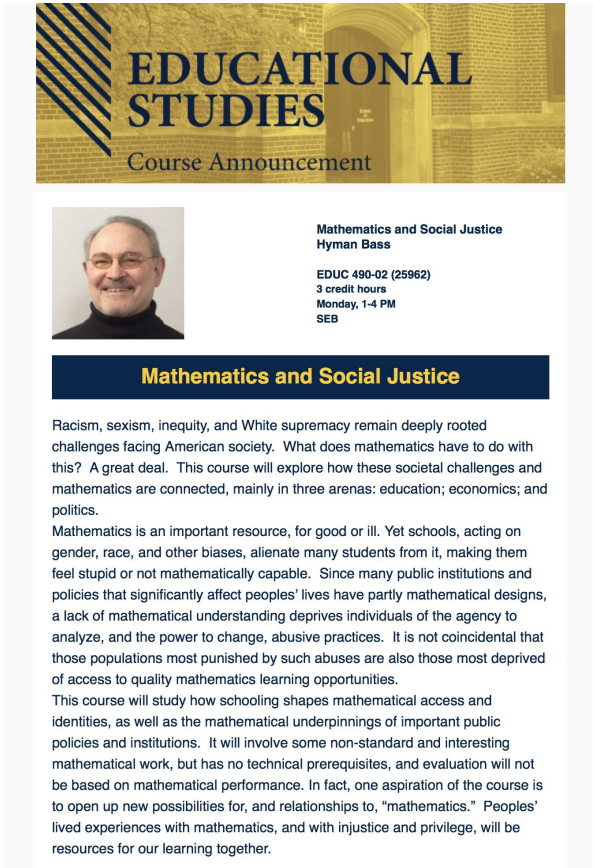
UNIVERSITY OF MICHIGAN

Who are we?



EDUCATIONAL STUDIES

Course Announcement

Mathematics and Social Justice
Hyman Bass

EDUC 490-02 (25962)
3 credit hours
Monday, 1-4 PM
SEB

Mathematics and Social Justice

Racism, sexism, inequity, and White supremacy remain deeply rooted challenges facing American society. What does mathematics have to do with this? A great deal. This course will explore how these societal challenges and mathematics are connected, mainly in three arenas: education; economics; and politics.

Mathematics is an important resource, for good or ill. Yet schools, acting on gender, race, and other biases, alienate many students from it, making them feel stupid or not mathematically capable. Since many public institutions and policies that significantly affect peoples' lives have partly mathematical designs, a lack of mathematical understanding deprives individuals of the agency to analyze, and the power to change, abusive practices. It is not coincidental that those populations most punished by such abuses are also those most deprived of access to quality mathematics learning opportunities.

This course will study how schooling shapes mathematical access and identities, as well as the mathematical underpinnings of important public policies and institutions. It will involve some non-standard and interesting mathematical work, but has no technical prerequisites, and evaluation will not be based on mathematical performance. In fact, one aspiration of the course is to open up new possibilities for, and relationships to, "mathematics." Peoples' lived experiences with mathematics, and with injustice and privilege, will be resources for our learning together.

We co-constructed and co-taught an undergraduate *Math & Social Justice* course at the University of Michigan School of Education.

Hy Bass - Professor of math & math education. Interested in math knowledge for teaching (MKT), math practices, and social justice

Elena Crosley - Doctoral student in mathematics education with prior degree in mathematics. Interested in students' mathematics identity and gender identity development.

Matt Dahlgren - Doctoral student in mathematics education. Prior degrees in mathematics. Interested in the mathematical work of teaching, social justice, and the nature of mathematics.

Our question...

How could we build a course

that explored connections between

mathematics and **social justice**

while upholding the integrity of **both**?

What do we mean by that?

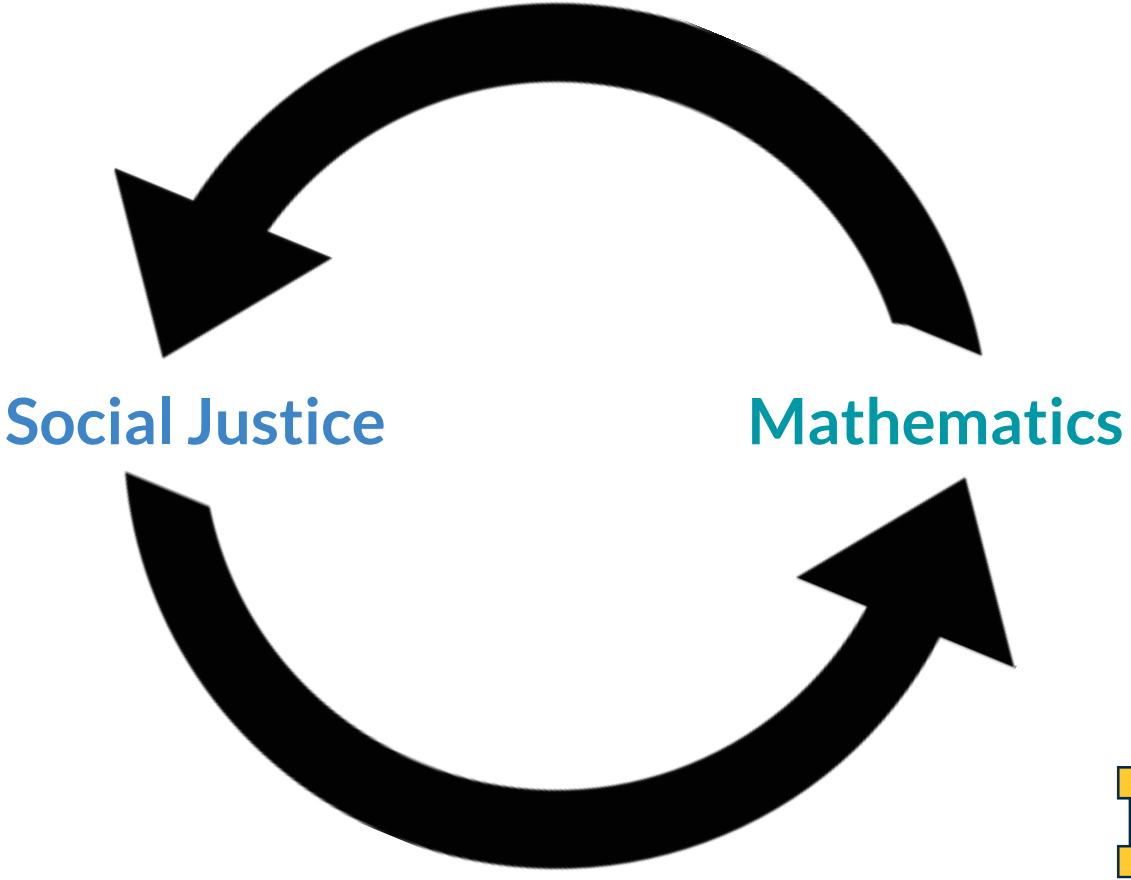
Mathematics

- Is not just focused on basic competency
- Offers a chance to explore, be creative, and test the limits of a representation
- Doesn't shy away from "serious" mathematical ideas

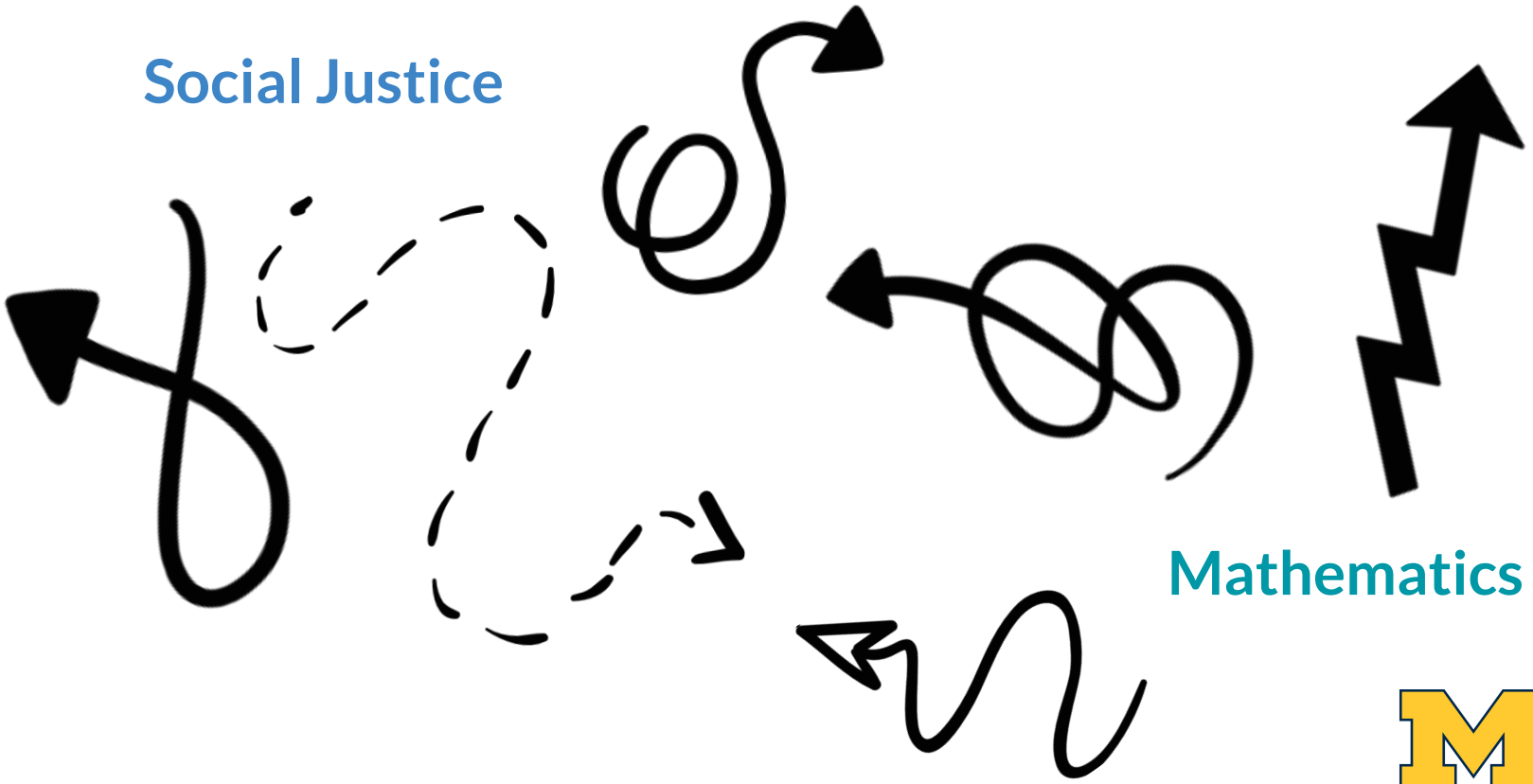
Social Justice

- About widening the perspective of the student, giving them some tools to understand the world, AND changing the system
- Not just talking about issues, but trying to find ways of addressing them
- Responsive to students
- Equitable enactment

How we hoped this would work



How it actually worked



A timeline of topics

Class 1

Birthday Problem, Probability, Voter Fraud, Voter Suppression

Class 2

Simpson's Paradox, Gender Discrimination, (Mis)representations of Data

Class 3

Pathways to Prison, Inequities in schools, Disrupting patterns in math class through better teaching

Class 4

Gerrymandering, How it happens, and Mathematical attempts at a solution

Class 5

Voting Methods, Social choice, Arrow Impossibility Theorem, the electoral college and apportionment

A timeline of topics

Class 6

Health Inequities, Mandatory Drug Screening of Welfare Recipients

Class 7

GPAs, Redlining, Interest Rates, Credit Cards, and Payday Loans

Class 8

Privacy, Spying, Encryption and Cryptography

Class 9

Fake News, Vulnerabilities to misinformation, Media Literacy

A timeline of topics

Class 10

Nature of Mathematics, Who can be a mathematician? What counts as math?

Class 11

Various images of mathematical activity, Danger of a Single Story

Class 12

School choice, Conjecture and proof in the classroom, Japanese mathematics lessons

Class 13

Mathematics at U-Mich, Vaccinations, Big data and insidious algorithms

Maybe it wasn't that bad after all

- Showcased a wide range of serious mathematical topics and social justice issues
- Made some interesting and unexpected connections between the two
- Delved deeply into topics in a variety ways
- Timely connections - "In the news"
- Demonstrated that mathematics could be used to expose and to disrupt inequities
- Dipped our toes into mathematically reimagining how things could be different.

Proof of concept?

- Despite the clunky and ad hoc nature of the course, in the end it did have a sort of coherence - informed citizenship through mathematics and better mathematics through informed citizenship.

- Maybe someone else might like to use this stuff

- The course didn’t have a tight structure, and felt at times like it was moving in a lot of different directions, but we already had a bunch of materials produced.

Returning to our question...

Revision:

How could we build ~~a course~~ self-contained units

that explored connections between

mathematics and **social justice**

while upholding the integrity of **both**?

Modules are born

- If we could could pull together several common threads from throughout the course then perhaps we could bootstrap the structure we were hoping for
- Possible outcomes of sharing our work:
  - Others might not have to go through the same difficult process we did
  - Materials might be used in new ways
- We applied for funding from the University's Diversity Innovation Fund and began expanding our lessons to be self-contained units organized by topic

Electoral Politics Package

Electoral Politics

This is a package of four Modules, each related to some aspect of electoral politics in the U.S.:

EP1. "Voter Fraud"
EP2. Apportionment
EP3. Gerrymandering
EP4. Social Choice & the Electoral College

Here we provide an overview of this package. A convenient background reference is the file called "The structure of the US government."

Voting: Principles; Methods; Abuses

Systems of Social Governance

Isolated individuals can (and must) make their own decisions. But societies, in order to gain the advantages of reasonably coherent collective behaviors, require some method of making societal decisions. There are, both theoretically and in historical reality, many ways to do this. At one extreme is an autocracy, in which all societal decisions reside in a single individual, or small group of individuals. At the opposite extreme is the

Democratic Principle:
Societal decisions should represent the collective will of all members of the society.

While the Democratic Principle has, on its face, a certain egalitarian appeal, figuring out how to implement it is a complex problem, facing both practical difficulties and some theoretical impossibilities (paradoxes)[1]. This is the domain of **Social Choice Theory**, that we shall discuss later, in Module EP4. It studies various models of voting used to express social preference.

Electoral Politics Background Info

The Constitutional Design of The Federal U. S. Government
From a Colony under the British Monarchy to A United, Democratic, Republic of States
An enduring tension: Federal vs. States' rights

Three ("Independent") Branches, with "Balance of Powers"

Legislative (bicameral **Congress**) Passes laws Can override presidential veto with a super majority Advice & Consent on Presidential Appointments		**Executive** Runs the government Departments & Agencies; Regulatory Power President can veto Laws	**Judicial** Appointed by president with advice & consent of senate
House (435 reps(*)) Authorizes Federal budget (proposed by President)	**Senate** (100 sens, 2/state) Advice & consent on pres. appointments	**President & Vice President**	**Federal Courts**
• **#State reps** is ~ proportional to state's population: • [State pop]/[US pop] x 435 (Not a whole number!) • 1 rep per **election district.** None for DC or territories	• 2 senators per state; None for DC or territories • 6 year terms • Statewide elections	• Elected by "**Electoral College.**" #Members: • 538 = 435 + 100 + 3 #House #Senate DC • Compromise between election by Congress and by **popular vote**. • Pres. & V.P. have 4-year terms. Now at most 2 terms.	Supreme court appointments for life.
Allocation is based on **national Census**, every 10 years. Issues: • How the **Census** is conducted (accuracy) • **Apportionment** of reps to states. Fraction problem.		• Individual voters vote for electors, not for president. • Hence <u>winner need not win popular vote</u>. • Each presidential candidate has a slate of electors for each state.	Federal laws & state laws (& courts) are sometimes in conflict (marijuana, immigration, voting, civil & religious rights, etc.)

A brief tour of a module: “Voter Fraud”

- Found that merely sharing slides/activities was not really sufficient.
  - Things that might be clear to us, could be less clear to others
  - Potential misuse
- What would be sufficient?
  - Quick Summary
  - Session Plan
  - Slides
  - Citations
  - Background literature for mathematics, history, politics, government, etc.
  - Extensions

Quick Summary of Module Content

Session 1 - "Voter Fraud"

Note:
In several modules we were forced to make a choice between presenting a coherent social justice narrative or covering additional mathematical topics. Thus, depending on the module, either mathematical content or social justice issues may be backgrounded while the other is foregrounded. Some sessions may seem mathematically light, but in these instances we have made efforts to include content which might broaden conceptions of what counts as mathematics.

What's "the matter"?
Voter fraud is an issue that has been alleged to be widespread by a number of politicians and organizations. Those who suggest that fraud is prevalent also frequently assert that such acts undermine our democratic process and therefore additional restrictions and monitoring are necessary. Those on the other side of the debate suggest that voter fraud is extremely rare and further assert that policies which aim to counteract voter fraud are actually just efforts to suppress votes in predominantly minority communities.

What's the context?
In the wake of the most recent presidential election, much attention has been paid to the question of alleged voter fraud. While the president's contention that millions of illegal votes were cast in favor of his opponent pushed the subject into the mainstream, assertions of voter fraud have been raised (by primarily Republican politicians) for a number of years. These assertions have led several states to begin implementing strategies that aim to curb alleged voter fraud. One such effort, the Interstate Voter Registration Crosscheck Program, seeks to address the issue of interstate double voting - which occurs when the same person votes in the same election in two different states. Crosscheck compares certain voter information (name and date of birth) from voter rolls in participating states, to check for instances of double voting. The program started in 2005 with just four participating states, but rapidly expanded under the direction of Kris Kolbach (the former Kansas Secretary of State and head of the now defunct Commission on Election Integrity) to include, at its height, thirty participating states. Numerous

Session Plan - Overview

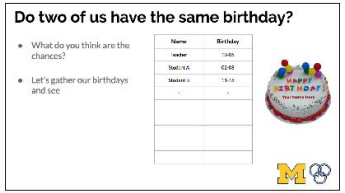
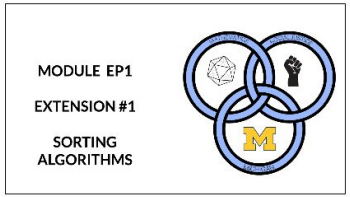
OVERVIEW OF EP1, SESSION 1

Instructional Segment	Time	Activities	Practices	Materials needed
Birthday Problem: "Do you think that two of us have the same birthday?"	5 mins (+20 min possible extension)	• Pose problem; speculate • Collectively tabulate data ○ Possible extension: Sorting Algorithms • Discuss • Introduce concept of probability (chance/likelihood)	• Working on mathematics collectively • Notice, and interpret, any unexpected patterns • Notice the sorting task (and its mathematization)	
Probability and Solution to Birthday Problem	15 mins	• Introduce probability formally • "Outcome space," " event space," notation • Discuss and derive some basic properties • Set up probability calculation in a simpler context • Wolfram Alpha Demonstration	• Mathematically defining and denoting intuitive concepts • Mathematically reasoning with these concepts • Applying them to new contexts	Wolfram Alpha CDF player and file Maybe deck of cards or a pair of dice
Birth*date* Problem / Modeling	15 mins (+15 min possible extension)	• Discuss extension to birthdate problem ○ Possible extension: Discussion of mathematical modeling • Affordances and constraints of computer simulations • Effect of non-uniform distributions (increases chance of match)	• Model with mathematics • Use appropriate tools strategically	McDonald & Levitt, 2008

Session Plan - Detail: Common Birthdays

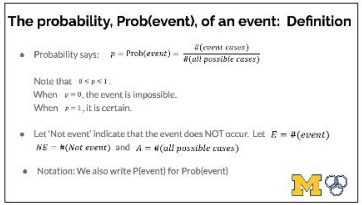
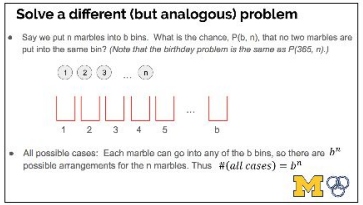
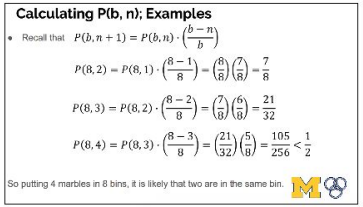
DETAILED LESSON PLAN - EP 1 SESSION 1

I. CLASS BIRTHDAYS: 5 mins (+20 mins for extension)

Slides	Detail	Commentary (Notes/Anticipations)
 	The class should begin with a question: *What do you think are the chances that two of us in this room share the same birthday?* Some possible ways to address this question: ● Construct a table with student names and birthdays to check. (slide # 2) ○ You can do this on a computer and project (or make poster version for the class to add to, if appropriate) ○ Each student can add their birthday to the table or you can do this ahead of time ● This could also be part of a larger "get to know you" activity if this module occurs fairly early on in the course ● *Optional Mathematical Extension:* Have the students line up in a random order and tell the person to the right of them their birthday. Then, depending on whose birthday is earlier, they switch spots and repeat the process until no further switching is possible. At the end of the activity, the earliest birthdays will be on one side and the latest birthdays will be on the other. ○ Possible questions to ask at the end: *How can we be sure this algorithm would work? Are there any*	If the sorting algorithms version of this activity is chosen, there are additional resources that might help you navigate the discussion that are worth looking at. Here are a few we recommend: - Visualizations and additional resources - Additional activity and resources - 5 minute video about three algorithms - Longer video about sorting algorithms

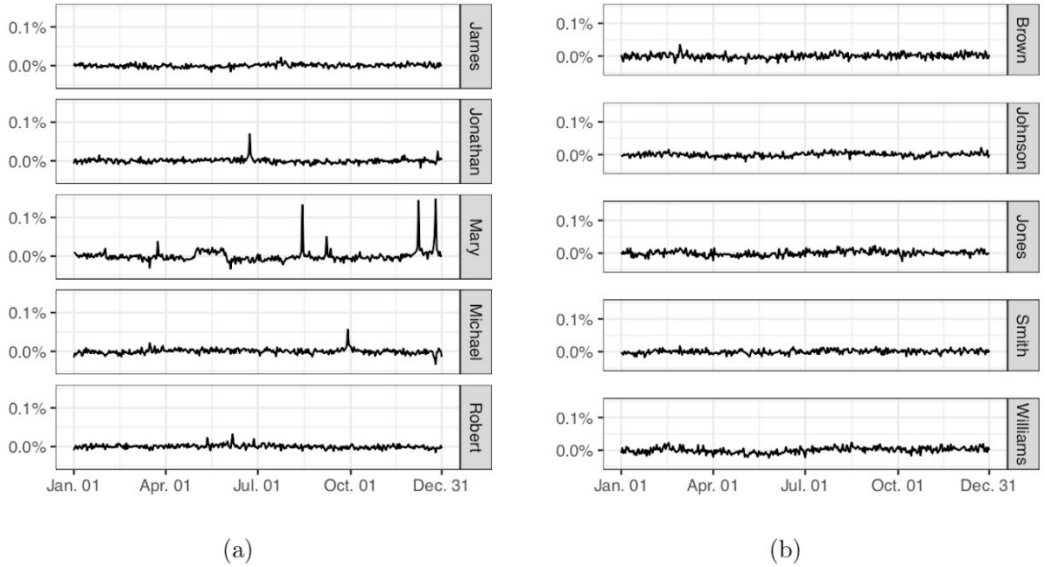
Session Plan - Detail: Probability

II. PROBABILITY & BIRTHDAY PROBLEM: 15 mins

Slides	Detail	Commentary (Notes/Anticipations)
	We have included some essential definitions and derivations of basic properties of probability in the lecture slides for this session. We advocate that the class work together (given the definitions) to collectively derive these properties through discussion.	
	When beginning the discussion of the birthday problem solution it may help to have a physical example (some manipulatives) of the analogous balls and bins problem available. Once the solution to this particular case has been fully articulated, the generalized version follows fairly straightforwardly (the generalized solution is located in the slides)	
	We also recommend computing some specific examples of the probability calculation to give some exposure to use of recursive formulas. In our course, we deferred the final calculation of the birthday problem to a computer after motivating it with the balls/bins problem (since the calculation for 365 "bins" is a bit unwieldy.) We have included a link to a handy interactive applet (from Wolfram Alpha) which graphs the probability of a birthday match as a function of the number of people. This can be used to both determine the correct answer (at 23 people, the probability	

Slides/Activities: Common Names

What's going on here?



Graphs from (Goel et al., 2017)

Citations & Resources

Session 1 - "Voter Fraud"
CITATIONS & RESOURCES

Information about Crosscheck and Kris Kobach:

1. https://www.nytimes.com/2017/06/13/magazine/the-man-behind-trumps-voter-fraud-obsession.html
    - Profile on Kris Kobach and his work related to voter ID laws, immigration law, questions of citizenship and overall voter restrictions.
2. https://www.rollingstone.com/politics/features/the-gops-stealth-war-against-voters-w435890
    - This article highlights some of the major issues with the Crosscheck Program used to disenfranchise legal voters. It specifically looks at Crosscheck data from Virginia, Georgia and Washington state. Rolling Stone journalists obtained a list of 1 million voters targeted by Crosscheck as double voters, and did an analysis of the list which demonstrated that it disproportionately threatened minority populations.
3. https://www.washingtonpost.com/news/wonk/wp/2017/07/20/this-anti-voter-fraud-program-gets-it-wrong-over-99-of-the-time-the-gop-wants-to-take-it-nationwide/?utm_term=.4e24959c770b
    - Discusses the birthday problem, McDonald and Levitt (2008), and Goel et al (2017) work in relation to the high number of false positives (as opposed to actual instances of double voting) which the Crosscheck Program returns.
4. http://www.kansascity.com/news/politics-government/article148434369.html
    - Covers news that Kobach prosecuted a ninth voter fraud case in Kansas and how he has continued to state that voter fraud is rampant. Article notes that Kobach has staunchly supported his claims that voter fraud is a huge problem despite providing any hard evidence.
5. http://kcur.org/post/kansans-caught-crosscheck-system-singled-out-kobachs-voter-fraud-campaign#stream/0

Mathematical Background Info

A Modest Introduction to (Discrete) Probability

Common Birthday

Ms. Sanchez greets her new high school class of 25 students, whom she has never met. After welcoming them she surprises them by saying, "I bet that two of you have the same birthday." Why would she think this? Some wonder if she might have some secret information about them, so they are reluctant to take on the bet. What does Ms. Sanchez know that prompts her to pose this bet?

This is a classic problem in probability theory, the theory that gives you tools to calculate the *likelihood* (or *chance,* or *probability*) of an *event*, (in this case the event that two people in the class have the same birthday). The likelihood, or probability, of an event E is a number, that we will denote P(E), between 0 and 1, with P(E) = 0 signifying that E can't happen, P(E) = 1 signifying that E will certainly happen, and P(E) > ½ signifying that the chances are better than even that E will happen. What Ms. Sanchez knows is that,

P(In a group of 25 people, two of them have the same birthday) > ½

How does she know this? More generally, how does one calculate P(E)?

Consider a simple example. In a bag of 100 marbles, 40 of them are red, and the other 60 are blue. If you randomly pick a marble from the bag, what are the chances that it is red? P(a randomly chosen marble is red) = 40/100 = 0.4 = 40%. In this fraction the denominator, 100, represents the total number of marbles that could be drawn (the "total possibility space" T), while the numerator, 40, represents the number of those among the possible marbles drawn that are red (the "event space" E). Thus, in general:

Extension: Sorting activities

Order up!

- Everyone please form a line at the front of the room.
- How can we sort ourselves so that the earliest birthday (January) is to the left and the latest birthday (December) is to the right?
- One method:
  - Turn to the person to the right of you and switch if your birthday is later than theirs.
  - Continue this until the person on your right has a later birthday than you.
  - Then start over again until everyone is sorted.
- Would this help us answer our birthday question?
- Let's try this out!

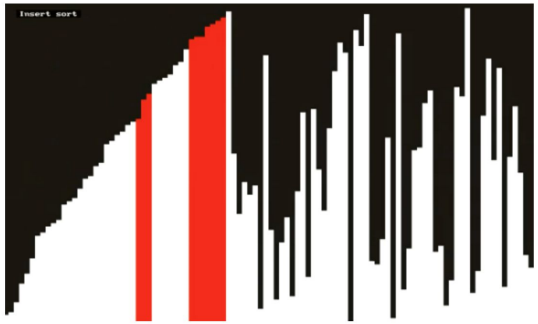
Order up! (Discussion)

- How can we be sure that we did that correctly?
- Were we missing any pairs?
- How do we know?
- How many comparisons did we make?
- Can anyone think of a way to do it faster?

Extension: Sorting Algorithms

TRY ME!

What different sorting algorithms sound like



GPA Activity

- You are frequently judged and compared competitively with others seeking the same goal.
  - Admission to college (College Board)
  - Grad School (GRE)
  - Seeking a job, or promotion
  - K-12 students, teachers, schools (NCLB tests and sanctions)
- Since these judgments are made on many people, they are based on mathematically designed measurements. Do these instruments give a valid measure of what is most important?
- How are values implicit in the ways that we are measured?

The Case of the GPA

(Small groups, 5-10 minutes)

- What is a GPA?
- What does it measure?
- And how?

Assumptions implicit in the way GPA is calculated

1. The difference between any two adjacent letter grades is the same.

2. Calculating GPA, we use the following scale: F = 0, D = 1, C = 2, B = 3, A = 4. That may seem like "the obvious way" to do it, and may be "the way we've always done it", but what does that choice imply?

Consider the case of Antwan and Ryan

Both have a B in one class and a D in another, for an overall GPA of (3 + 1)/2 = 2 (this would be a C)

New Information

Right before the end of the semester, Antwan raises his B to an A, while Ryan raises his D to a C.

- What are their new GPAs?
  - Antwan and Ryan still have the same GPA, since (4+1)/2 = (3+2)/2 = 2.5. (This would be a B-)
  - However, that implies that we've decided that having a B and a C is exactly as good as having an A and a D.
- What do you think about that? Is it reasonable for someone with a B and a C and someone with an A and a D to have the same GPA? Why or why not?

  (Talk to a partner.)

Hypotheticals

- What if I don't accept the contention that the difference between any two adjacent grades should be the same? What if, instead, I suggest that Fs and Ds are to be avoided, while C, "average", is an important level to meet or exceed.
  - Meanwhile, B is a decent step above C, but from B, "good", to A, "excellent", is less of a leap.
- What if, then, we instead had the following values for GPA:

  F = 0, D = 1, C = 4, B = 6, A = 7 ?
- Think about implications for this sort of GPA system, and then share ideas with the class.
  - For example, such a system would likely cause students to put much more work time into courses where they have a grade on the lower end of a large divide, such as a D, than to put that work into courses where their grade is one with a small gap ahead of it, such as a B.

Antawn & Ryan - Part II

Recalculate Antwan's and Ryan's GPAs before and after they raise their grades, using the new system.

Solution: They both start out with a B and a D, which now gives them a 3.5, or slightly below a C average, but Antwan ends with an A and a D, which gives him a 4.0, a C exactly, while Ryan has a B and a C, which gives him a 5.0, a full point above a C (and a full point below a B).

Is this way of calculating GPA more, or less, fair?

Form Small Groups

- What do you think is the most fair way to calculate GPA?
- Should a person with all Cs have the same GPA as a person with 3 As and 3 Fs? If not, who should have a higher or lower GPA?
- How should things like AP credit be treated in this new system?
- What different outcomes as far as how students prioritize classes and approach schoolwork might arise because of this new system? Are those outcomes that fit with your values?

What does this have to do with social justice?

Take a minute or two to make a journal entry about this discussion and your thoughts.

Discussion

- In what ways can mathematics and social justice be authentically integrated in an educational context?
- How have others navigated this tension between mathematics and social justice?
- Why is work like this important? At this moment?
- How and where can this conversation be continued?
- Mathematizing example