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In this first chapter we learn how to think about questions that need only good enough answers. We find those answers with quick estimates that start with reasonable assumptions and information you have at your fingertips. To make the arithmetic easy we round numbers drastically and count zeroes when we have to multiply.

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