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Calculating on the Back of an Envelope

In this first chapter we learn how to think about questions that need only good enough answers. We find those answers with quick estimates that start with reasonable assumptions and information you have at your fingertips. To make the arithmetic easy we round numbers drastically and count zeroes when we have to multiply.

Chapter goals

Goal 1.1. Verify quantities found in the media, by checking calculations and with independent web searches.

Goal 1.2. Estimate quantities using common sense and common knowledge.

Goal 1.3. Learn about the Google calculator (or another internet calculator).

Goal 1.4. Round quantities in order to use just one or two significant digits.

Goal 1.5. Learn when not to use a calculator—become comfortable with quick approximate mental arithmetic.

Goal 1.6. Work with large numbers.

Goal 1.7. Work with (large) metric prefixes

Goal 1.8. Use straightforward but multi-step conversions to solve problems.

1.1 Billions of phone calls?

On September 20, 2007, the United States House of Representatives Permanent Select Committee on Intelligence met to discuss legislation (the Protect America Act) that expanded the government's surveillance powers. The committee was concerned with the balance between protecting the country and preserving civil liberties. They questioned Admiral Michael McConnell, Director of National Intelligence, about governmental monitoring of international telephone calls. In the course of the hearing, Admiral McConnell said that he did not know how many Americans' telephone conversations may have been overheard through US wiretaps on foreign phone lines saying "I don't have the exact number … considering there are billions of transactions every day." [R3]

McConnell knows he's not reporting an exact number. Is his claim that there are "billions of transactions" a genuine estimate, or just a way of saying "lots and lots of transactions?" We can find out using just a little arithmetic and a little common sense.

In 2007 there were about 300 million people in the United States. There are more now, but 300 million is still often a good approximation for back-of-an-envelope calculations.

If everyone in the United States talked daily on a foreign phone call that would come to about 300 million calls. That's 300,000,000: 3 with eight zeroes. McConnell talked about "billions." He didn't say how many, but "billion" means nine zeroes, which is ten times as large. To get to billions of phone calls in a day each person in the country would have to make ten of them. That doesn't seem to make sense.

So when McConnell says "I don't have the exact number" he probably does mean just lots and lots.

We didn't need pencil and paper, let alone a calculator, to do the arithmetic—we needed one fact (the population of the United States), and then simply counted zeroes.

But before accusing McConnell of fudging the numbers, we should examine our own assumptions. Several years after this hearing *The Washington Post* reported that

> Every day the National Security Agency intercepts and stores *1.7 billion* international e-mails, phone calls, texts and other communications. [R4]

So in 2010 there were indeed billions of transactions each day if email and other electronic communications are counted along with telephone calls. There were probably fewer in 2007, quite probably still billions.

In the spring of 2013 this issue received new media attention when the extent of NSA data collection became public. The issue then wasn't intercepting and storing international communications, it was collecting and storing *metadata* about domestic telephone calls: who called whom, and when, although not what was said.

1.2 How many seconds?

Have you been alive for a thousand seconds? a million? a billion? a trillion?

Before we estimate, what's your guess? Write it down, then read on.

To check your estimate, you have to do some arithmetic. There are two ways to go about the job. You can start with seconds and work up through hours, days and years, or start with thousands, millions and billions of seconds and work backwards to hours, days and years. We'll do it both ways.

How many seconds in an hour? Easy: $60 \times 60 = 3600$. So we've all been alive much more than thousands of seconds.

Before we continue, we're going to change the rules for arithmetic so that we can do all the multiplication in our heads, without calculators or pencil and paper. We will round numbers so that they start with just one nonzero digit, so 60×60 becomes 4000. Of course we can't say $60 \times 60 = 4{,}000$; the right symbol is $\approx$, which means "is approximately." Then an hour is

$$60 \times 60 \approx 4000$$

seconds.

There are 24 hours in a day. $4 \times 24 \approx 100$, so there are

$$4{,}000 \times 24 \approx 100{,}000$$

seconds in a day.

Or we could approximate a day as 20 hours, which would mean (approximately) 80,000 seconds. We'd end up with the same (approximate) answer.

Since there are about a hundred thousand seconds in a day, there are about a million seconds in just 10 days. That's not even close to a lifetime, so we'll skip working on days, weeks or months and move on to years.

How many seconds in a year? Since there are (approximately) 100,000 in a day and (approximately) 400 days in a year there are about 40,000,000 (forty million) seconds in a year.

If we multiply that by 25 the 4 becomes 100, so a 25-year old has lived for about 1,000,000,000 (one billion) seconds.

Does this match the estimate you wrote down for your lifetime in seconds?

The second way to estimate seconds alive is to work backwards. We'll write the time units using fractions—that's looking ahead to the next chapter—and round the numbers whenever that makes the arithmetic easy. Let's start with 1000 seconds.

$$\begin{aligned}
1000\ \cancel{\text{seconds}} \times \frac{1\ \text{minute}}{60\ \cancel{\text{seconds}}} &= \frac{1000}{60}\ \text{minutes} \\
&= \frac{100}{6}\ \text{minutes (cancel a 0)} \\
&= \frac{50}{3}\ \text{minutes (cancel a 2)} \\
&\approx \frac{60}{3}\ \text{minutes (change 50 to 60—make division easy)} \\
&= 20\ \text{minutes.}
\end{aligned}$$

We're all older than that.

How about a million seconds? A million has six zeroes—three more than 1,000, so a million seconds is about 20,000 minutes. Still too many zeroes to make sense of, so convert to something we can understand—try hours.

$$20{,}000\ \cancel{\text{minutes}} \times \frac{1\ \text{hour}}{60\ \cancel{\text{minutes}}} = \frac{20{,}000}{60}\ \text{hours} = \frac{1{,}000}{3}\ \text{hours} \approx 300\ \text{hours.}$$

There are 24 hours in a day. To do the arithmetic approximately use 25. Then $300/25 = 12$ so 300 hours is about 12 days. We've all been alive that long.

How about a billion seconds? A billion is a thousand million, so we need three more zeroes. We can make sense of that in years:

$$12{,}000\ \cancel{\text{days}} \times \frac{1\ \text{year}}{365\ \cancel{\text{days}}} = \frac{12{,}000}{365}\ \text{years} \approx \frac{12{,}000}{400}\ \text{years} \approx 30\ \text{years.}$$

Since a billion seconds is about 30 years, it's in the right ballpark for the age of most students.

A trillion is a thousand billion—three more zeroes. So a trillion seconds is about 30,000 years. Longer than recorded history.

1.3 Heartbeats

In *The Canadian Encyclopedia* a blogger noted that

> The human heart expands and contracts roughly 100,000 times a day, pumping about 8,000 liters of blood. Over a lifetime of 70 years, the heart beats more than 2.5 billion times, with no pit stops for lube jobs or repairs. [R5]

Should we believe "100,000 times a day" and "2.5 billion times in a lifetime"?

If you think about the arithmetic in the previous section in a new way, you may realize you've already answered this question. Since your pulse rate is about 1 heartbeat per second, counting seconds and counting heartbeats are different versions of the same problem. We discovered that there are about 100,000 seconds in a day, so the heartbeat count is about right. We discovered that 30 years was about a billion seconds, and 70 is about two and a half times 30, so 70 years is about 2.5 billion seconds. Both the numbers in the article make sense.

Even if we didn't know whether 100,000 heartbeats in a day was the right number, we could check to see if that number was consistent with 2.5 billion in a lifetime. To do that, we want to calculate

$$100{,}000\frac{\text{beats}}{\text{day}} \times 365\frac{\text{days}}{\text{year}} \times 70\frac{\text{years}}{\text{lifetime}}.$$

Since we only need an approximate answer, we can simplify the numbers and do the arithmetic in our heads. If we round the 365 up to 400 then the only real multiplication is $4 \times 7 = 28$. The rest is counting zeroes. There are eight of them, so the answer is approximately $2{,}800{,}000{,}000$. That means the 2.5 billion in the article is about right. Our answer is larger because we rounded up.

The problems we've tackled so far don't have exact numerical answers of the sort you are used to. The estimation and rounding that goes into solving them means that when you're done you can rely on just a few *significant digits* (the digits at the beginning of a number) and the number of zeroes. Often, and in these examples in particular, that's all you need. Problems like these are called "Fermi problems" after Enrico Fermi (1901–1954), an Italian physicist famous (among other things) for his ability to estimate the answers to physical questions using very little information.

1.4 Calculators

The thrust of our work so far has been on mental arithmetic. You can always check yours with a calculator. You probably have one on the phone in your pocket. There's one on your computer. But those require pressing keys or clicking icons. If you have internet access, Google's is easier to use—simply type

```
100,000 * 365 * 70
```

into the search box. Google displays a calculator showing

```
2555000000
```

.

That 2.555 billion answer is even closer than our first estimate to the 2.5 billion approximation in the article. The Bing search engine offers the same feature.

You can click on the number and operation keys in the Google calculator to do more arithmetic. Please don't. Just type an expression in the Google search bar. Stick to the keyboard rather than the mouse. It's faster, and you can fix typing mistakes easily.

The Google calculator will do more than just the arithmetic—it can keep track of units. Although it doesn't deal with heartbeats, it does know about miles, and speeds like miles per day and miles per year. We can make it do our work for us by asking about miles instead of heartbeats. Search for

100,000 miles per day in miles per 70 years

and Google rewards you with

100000 (miles per day) = 2.55669539 × 10^9 miles per (70 years)

.

The "$\times 10^9$" means "add nine zeroes" or, in this case, "move the decimal point nine places to the right", so

$$\begin{aligned} 100000 \text{ (miles per day)} &= 2.55669539 \times 10^9 \text{ miles per (70 years)} \\ &\approx 2.6 \text{ billion miles per (70 years).} \end{aligned}$$

That is again "more than 2.5 billion."

The exact answer from Google is even a little more than the 2,555,000,000 we found when we did just the arithmetic since Google knows a year is a little longer than 365 days—that's why we have leap years.

So 100,000 heartbeats per day does add up to about 2.5 billion in 70 years. We've checked that the numbers are consistent—they fit together.

But are they correct? Does your heart beat 100,000 times per day? To think sensibly about a number with lots of zeroes we can convert it to a number of something equivalent with fewer zeroes—in this case, heartbeats per minute. That calls for division rather than multiplication:

$$100{,}000 \frac{\text{beats}}{\text{day}} \times \frac{1 \text{ day}}{24 \text{ hours}} \times \frac{1 \text{ hour}}{60 \text{ minutes}}.$$

To do the arithmetic in your head, round the 24 to 25. Then $25 \times 6 = 150$—there are about 1,500 minutes in a day. Then $100{,}000/1{,}500 = 1{,}000/15$. Since $100/15$ is about 7, we can say that $1{,}000/15$ is about 70. 70 beats per minute is a reasonable estimate for your pulse rate, so 100,000 heartbeats per day is about right.

Google tells us

100 000 (miles per day) = 69.4444444 miles per minute

.

The nine and all the fours in that 69.4444444 are much too precise. The only sensible thing to do with that number is to round it to 70—which is what we discovered without using a calculator.

Sometimes even the significant digits can be wrong and the answer right, as long as the number of zeroes is correct. Informally, that's what we mean when we say the answer is "*in the right ballpark*." The fancy way to say the same thing is "the *order of magnitude* is correct." For example, it's right to say there are hundreds of days in a year—not thousands, not tens. There are billions (nine zeroes) of heartbeats in a lifetime, not hundreds of millions (eight zeroes), nor tens of billions (ten zeroes).

1.5 Millions of trees?

On May 4, 2010 Olivia Judson wrote in *The New York Times* [R6] about Baba Brinkman, who describes himself on his web page as

> …a Canadian rap artist, playwright, and former tree-planter who worked in the Rocky Mountains every summer for over ten years, personally planting more than one million trees. He is also a scholar with a Masters in Medieval and Renaissance English Literature. [R7]

How long would it take to personally plant a million trees? Is Brinkman's claim reasonable?

To answer that question you need two estimates—the time it takes to plant one tree, and the time Brinkman may have spent planting. We have some information about the second of these—more than ten summers.

To plant a tree you have to dig a hole, put in a seedling and fill in around the root ball. It's hard to imagine you can do that in less than half an hour.

If Brinkman worked eight hours a day he would plant 16 trees per day. Round that up to 20 trees per day to make the arithmetic easier and give him the benefit of the doubt. At that rate it would take him $1{,}000{,}000/20 = 50{,}000$ days to plant a million trees. If he planted trees 100 days each year, it would take him 500 years; if he planted trees for 200 days out of the year, it would take him 250 years. So his claim looks unreasonable.

What if we change our estimates? Suppose he took just ten minutes to plant each tree and worked fifteen hour days. Then he could plant nearly 100 trees per day. At that rate it would take him 10,000 days to plant a million trees. If he worked 100 days each summer he'd still need about 100 years. That's still much more than the "more than 10 years" in the quotation. So on balance we believe he's planted lots of trees, but not "personally . . . more than one million."

It's the "personally" that makes this very unlikely. We can believe the million trees if he organized tree-planting parties, perhaps with people manning power diggers of some kind. Or if planting acorns counted as planting trees.

This section, first written in 2010, ended with that unfunny joke until 2013, when Charles Wibiralske, teaching from this text, wondered if we might be overestimating the time it takes to plant a tree. To satisfy his curiosity, he found Brinkman's email address and asked. The answer was a surprising (to him, and to us) ten seconds! So our estimate of 10 minutes was 60 times too big. That means our 100-year estimate should really have been only about two years! That's certainly possible. If it took him a minute per tree rather than 10 seconds he could still have planted a million trees in ten summers.

Brinkman tells the story of Wibiralske's question and this new ending in his blog at `www.bababrinkman.com/insult-to-injury/`. When you visit you can listen to "The Tree Planter's Waltz" (`www.youtube.com/watch?v=jk-jifbpcww`).

The moral of the story: healthy skepticism about what you read is a good thing, as long as you're explicit and open minded about the assumptions you make when you try to check. That's a key part of using common sense.

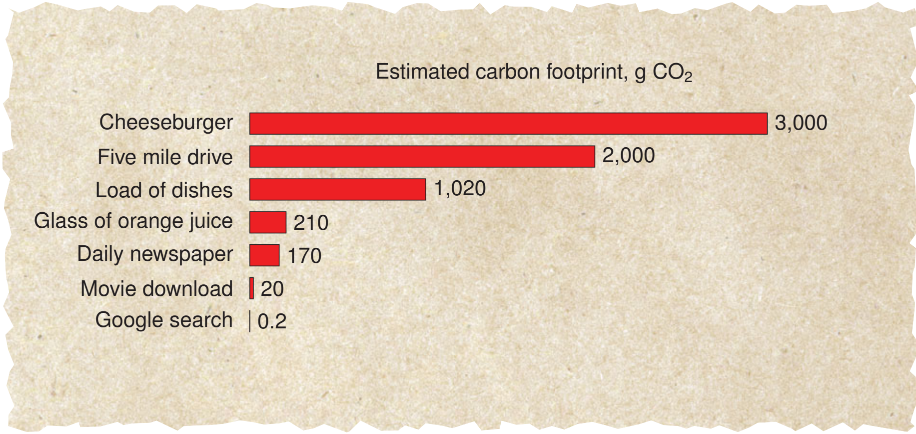

Figure 1.1. Carbon footprints [R8]

Brinkman's blog ends this way:

> Hurray! In the end it's a classic example of ...the drunkard's walk towards knowledge. When our views are self-correcting and open to revision based on new evidence, they will continue to hone in on increasingly accurate representations of the real world. That's good honest skepticism, and when it wins over bad, knee-jerk, "it's hard to imagine" skepticism, that's a beautiful thing.

1.6 Carbon footprints

Discussions about global warming and climate change sometimes talk about the *carbon footprint* of an item or an activity. That's the total amount of carbon dioxide (CO_2) the item or activity releases to the atmosphere. An article in *The Boston Globe* on October 14, 2010 listed estimates of carbon footprints for some common activities. Among those was the 210 gram carbon footprint of a glass of orange juice. That includes the carbon dioxide cost of fertilizing the orange trees in Florida, harvesting the oranges and the carbon dioxide generated burning oil or coal to provide energy to squeeze the oranges, concentrate and freeze the juice and then ship it to its destination. It's just an estimate, like the ones we're learning to make, but much too complex to ask you to reproduce. We drew Figure 1.1 using the rest of the data from the article.

Let's look at the orange juice. How many glasses are consumed in the United States each day? If we estimate that about 5% of the 300 million people in the country have orange juice for breakfast that means 15 million glasses. So the ballpark answer is on the order of 10 to 20 million glasses of orange juice. We can use that to estimate the total orange juice carbon footprint: each glass contributes 200 grams, so 10 to 20 million glasses contribute 2 to 4 billion grams each day.

It's hard to imagine 2 billion grams. You may have heard of kilograms —a kilogram is about two pounds. In the metric system *kilo* means "multiply by 1,000" so a kilogram is 1,000 grams. Then 2 billion grams of carbon is just 2 million kilograms. That's about 4 million pounds, or 2 thousand tons.

Since there are seven activities listed in the graphic, there are six other Fermi problems like this one for you to work on:

- Google search
- Movie download
- Daily newspaper printed
- Dishwasher run
- Five miles driven
- Cheeseburger consumed

For each you can estimate the total number of daily occurrences and then the total daily carbon contribution. We won't provide answers here because we don't want to spoil a wonderful class exercise.

1.7 Kilo, mega, giga

Counting zeroes is often best done three at a time. That's why we separate groups of three digits by commas. Each step from thousands to millions to billions adds three zeroes. The metric system has prefixes for that job.

We've seen that *kilo* means "multiply by 1,000". Similarly, *mega* means "multiply by 1,000,000". A Megabucks lottery prize is millions of dollars. A megathing is 1,000,000 things, whatever kind of thing you are interested in.

New Hampshire's Seabrook nuclear power plant is rated at 1,270 megawatts. So although you may not know what a watt is, you know this power plant can generate 1,270,000,000 of them. The symbols for "mega" and for "watt" are "M" and "W" so you can write 1,270 megawatts as 1,270 MW.

How many 100 watt bulbs can Seabrook light up? Just take the hundred's two zeroes from the mega's six, leaving four following the 1,270. Putting the commas in the right places, that means 12,700,000 bulbs. Or, if you want to be cute, about 13 megabulbs.

Next after mega is *giga*: nine zeroes. The symbol is "G". When you say "giga" out loud the G is hard, even though it's soft in the word "gigantic".

You could describe Seabrook as a 1.27 gigawatt power plant.

Table 1.2 describes the metric prefixes bigger than giga. There's no need to memorize it. You will rarely need the really big ones. You can look them up when you do.

The metric system also has prefixes for shrinking things as well as these for growing them. Since division is harder than multiplication, we'll postpone discussing those prefixes until we need them, in Section 2.7.

1.8 Exercises

Notes about the exercises:

- The preface has information about the exercises and the solutions; we suggest you read it.
- One of the ways to improve your quantitative reasoning skills is to write about what you figure out. The exercises give you many opportunities to practice that. The answer to a question should be more than just a circle around a number or a simple "yes" or "no". Write complete

Table 1.2. Metric prefixes

Name	Symbol	Meaning	English name	Zeroes
Kilo	K	$\times 10^3$	thousand	3
Mega	M	$\times 10^6$	million	6
Giga	G	$\times 10^9$	billion	9
Tera	T	$\times 10^{12}$	trillion	12
Peta	P	$\times 10^{15}$	quadrillion	15
Exa	E	$\times 10^{18}$	quintillion	18
Zetta	Z	$\times 10^{21}$	sextillion	21
Yotta	Y	$\times 10^{24}$	septillion	24

paragraphs that show your reasoning. Your answer should be complete enough so that you can use a corrected homework paper to study for an exam without having to go back to the text to remember the questions.

- It often helps to write about doubt and confusion rather than guessing at what you hope will turn out to be a right answer.
- To solve Fermi problems you make assumptions and estimates. It's those skills we are helping you develop. The web can help, but you should not spend a lot of time searching for answers to the particular questions we ask. That's particularly true since many of the problems don't have a single right answer.
- Be sure to make your estimates and assumptions explicit. Then you can change them easily if necessary, as we did in Section 1.5.

Your instructor has a Solutions Manual with answers to the exercises, written the way we hope you will write them. Ask him or her to provide some for you.

Exercise 1.8.1. [S][Section 1.1] Warren Buffet is very rich.

In *The Boston Globe* on September 30, 2015 you could read that Warren Buffet is worth $62 billion, and that

> . . . if [he] gave up on aggressive investing and put his money into a simple savings account, with returns at a bare 1 percent, he'd earn more in interest each hour than the average American earns in a year. [R9]

Use the data in this story to estimate the annual earnings of the average American. Do you think the estimate is reasonable?

Exercise 1.8.2. [S][Section 1.1][Goal 1.1][Goal 1.2][Goal 1.4] Dropping out.

In his May 17, 2010 op-ed column in *The New York Times* Bob Herbert noted that the dropout rate for American high school students was one every 26 seconds. [R10]

Is this number reasonable?

Exercise 1.8.3. [S][Section 1.1][Goal 1.1][Goal 1.2][Goal 1.5][Goal 1.8] Bumper sticker politics.

In the fall of 2015 at donnellycolt.com you could buy a button that said

> Every Minute 30 Children Die of Hunger and Inadequate Health Care While the World Spends $1,700,000 on War

or a small vinyl sticker with the claim

> Every Minute the World Spends $700,000 on War While 30 children Die of Hunger & Inadequate Health Care. [R11]

What are these items trying to say? Do the numbers make sense?

Your answer should be a few paragraphs combining information you find on the web (cite your sources—how do you know they are reliable?) and a little arithmetic.

Exercise 1.8.4. [S][Section 1.1][Goal 1.1][Goal 1.2][Goal 1.4][Goal 1.8] Two million matzoh balls.

In 2012 a modest restaurant in Newton Centre, MA advertised

> *Johnny's Luncheonette*
> *Over 2 Million*
> *Matzoh Balls*
> *served!*

A year or so later students thinking about whether it was reasonable found that Johnny's web site at www.johnnysluncheonette.com/ claimed "Over 1 Million Matzoh Balls Served!"

What would you believe?

[See the back of the book for a hint.]

Exercise 1.8.5. [S][Section 1.1][Goal 1.2][Goal 1.5] *Writing Your Dissertation in Fifteen Minutes a Day*.

Joan Bolker's book with that title sold about 120,000 copies in the first fifteen years since its publication in 1998.

Estimate the fraction of doctoral students who bought this book.

Exercise 1.8.6. [S][W][Section 1.1][Goal 1.1][Goal 1.2][Goal 1.4][Goal 1.5] Smartphone apps may help retail scanning catch on.

Some grocery stores are experimenting with a new technology that allows customers to scan items as they shop. Once the customer is done, he or she completes the transaction online and never has to stand in the checkout line. On March 11, 2012 *The Boston Globe* reported that

> Modiv Media's scan-it-yourself technology [is installed] in about 350 Stop & Shop and Giant stores in the United States. Many consumers have embraced the system; Stop & Shop spokeswoman Suzi Robinson said the service handles about one million transactions per month. [R12]

(a) Estimate the number of customers per day per store who use this self-scanning technology.
(b) Estimate the number of customers per day per store.
(c) Estimate the percentage of customers who use the technology.

Exercise 1.8.7. [S][Section 1.1][Goal 1.1][Goal 1.2][Goal 1.4][Goal 1.5] Health care costs for the uninsured.

On March 22, 2012 Linda Greenhouse wrote in *The New York Times* that the average cost of a family insurance policy increased by $1,000 a year because health care providers needed to recover $43 billion annually for health care costs of the uninsured. [R13]

(a) What it the annual cost per United States resident for medical care for the uninsured?
(b) Estimate (or research) how much an average person spends each year buying food at the grocery store (do not include restaurant purchases). Compare your answer to your answer from the previous question about the cost of medical care.
(c) Estimate (or research) the number of uninsured people, and then estimate the cost per uninsured person for medical care. Does your answer make sense to you?

Exercise 1.8.8. [S][Section 1.2][Goal 1.1] Is 25 the same as 30?

In Section 1.2 we showed that a 25 year old has lived for about a billion seconds. Then we estimated that a billion seconds is about 30 years.

(a) Explain why we got two different answers—25 and 30 years.
(b) Are the answers really different?
(c) Compare them to what the Google calculator says about a billion seconds.

Exercise 1.8.9. [S][Section 1.2][Goal 1.1][Goal 1.2][Goal 1.4][Goal 1.5] Spoons around the world.

According to the website `www.majorwastedisposal.com/trashtrivia.html`, Americans throw out enough plastic utensils (knives, forks and spoons) every year to circle the equator 300 times.

(a) Use the information stated at the beginning of the problem to estimate the average number of plastic utensils each person throws out each year.
(b) Is the assertion reasonable?

Exercise 1.8.10. [S][Section 1.2][Goal 1.1][Goal 1.2][Goal 1.3] 1,000,000,000,000,000,000,000,000,000.

On July 25, 2010 Christopher Shea wrote in *The Boston Globe* about "hella", a new metric prefix popular among geeks in northern California.

> Austin Sendek, a physics major at the University of California Davis, wants to take "hella" from the streets and into the lab. With the help of a Facebook-driven public relations campaign, he's petitioning the Consultative Committee on Units, a division of the very serious Bureau International des Poids et Mesures, to anoint "hella" as the official term for a previously unnamed, rather large number: 10 to the 27th power. (The diameter of the universe, by Sendek's reckoning, is 1.4 hellameters.) [R14]

(a) Does the Google calculator know about hella?
(b) Does the Bing search engine know about hella?

Exercise 1.8.11. [S][Section 1.2][Goal 1.2][Goal 1.4][Goal 1.5] Counting fish.

In *To the Top of the Continent* Frederick Cook wrote about this incident in his Alaska travels.

> The run of the hulligans was very exciting … Mr. Porter's thoughts ran to mathematics, he figured that the train of hulligans was twelve inches wide and six inches deep and that it probably extended a hundred miles. Estimating the number of fish in a cubic foot at ninety-one and one half, he went on to so many millions that he gave it up, suggesting that we try and catch some. [R15]

(a) How many millions of hulligans did Mr. Porter try to count?
(b) What's wrong with the precision in this paragraph?
(c) What's a hulligan? Cook's book was published in 1908. Are there any hulligans around today?

Exercise 1.8.12. [S][Section 1.2][Goal 1.1] Millions jam street-level crime map website.

In early 2011, the British government introduced a crime-mapping website that allows people to see crimes reported by entering a street name. The launch of the site, however, was problematic. The BBC reported that the web site was jammed by up to five million hits per hour, about 75 thousand a minute. [R16]

(a) Are the figures "five million an hour" and "75,000 a minute" consistent?
(b) Estimate the fraction of the population of London trying to look at that web site. Does your answer make sense?

[See the back of the book for a hint.]

Exercise 1.8.13. [S][Section 1.2][Goal 1.4][Goal 1.5][Goal 1.8] The popularity of social networks.

From *The New York Times*, June 28, 2011:

> In May [2011], 180 million people visited Google sites, including YouTube, versus 157.2 million on Facebook, according to comScore. But Facebook users looked at 103 billion pages and spent an average of 375 minutes on the site, while Google users viewed 46.3 billion pages and spent 231 minutes. [R17]

(a) How many webpages did the average Facebook user visit? How many webpages did the average Google user visit?
(b) On average, how many webpages per day in May did a Facebook user visit? Compare this to the average number of webpages per day for the Google user.
(c) On average, who spent more time on each page, the Facebook user or the Google user?
(d) Think about your own web behavior. Do these numbers seem reasonable to you?

Exercise 1.8.14. [S][Section 1.2][Goal 1.1][Goal 1.2][Goal 1.5][Goal 1.8] How rich is rich?

On September 19th, 2011 Aaron S. from Florida posted a comment at *The New York Times* in which he claims that if you left one of the 400 wealthy people with just a billion dollars he could not spend his fortune in 30 years at a rate of $100,000 a day. [R18]

Is Aaron's arithmetic right? Can you do this calculation without a calculator? Without pencil and paper?

Exercise 1.8.15. [S][Section 1.2][Goal 1.1][Goal 1.2][Goal 1.5][Goal 1.8] Greek debt.

In her September 26, 2011 *New York Times* review of Michael Lewis's book *Boomerang* Michiko Kakutani noted that Lewis said Greek debt of $1.2 trillion amounted to about $250,000 for each working Greek. [R19]

(a) Use Lewis's statement to estimate the population of working Greeks at the time he wrote the article.
(b) Use the web to find the national debt and population of Greece (for 2011 if you can, now if you can't).
(c) Do the answers to the previous two parts of this exercise agree? If not, what might explain any differences?
(d) Compare Greek per capita national debt to that in the United States.
(e) Here's a political question: is large national debt a bad thing? You can find both "yes" and "no" answers on the web. Here's one place to start: `www.npr.org/templates/story/story.php?storyId=99927343`.

Exercise 1.8.16. [S][Section 1.3][Goal 1.1][Goal 1.4][Goal 1.6] No Lunch Left Behind.

From *The New York Times*, Feb 20, 2009, in a column by Alice Waters and Katrina Heron with that headline:

> How much would it cost to feed 30 million American schoolchildren a wholesome meal? It could be done for about $5 per child, or roughly $27 billion a year, plus a one-time investment in real kitchens. [R20]

There are three numbers in the paragraph. Are they reasonable? Are they consistent with each other and with other numbers you know?

Exercise 1.8.17. [S][Section 1.3][Goal 1.2][Goal 1.4][Goal 1.5] Brush your teeth twice a day—but turn off the water.

The Environmental Protection Agency says on its website that

> You can save up to 8 gallons of water by turning off the faucet when you brush your teeth in the morning and before bedtime. [R21]

(a) Estimate how much water a family of four would use each week, assuming they left the water running while brushing.
(b) Estimate how much water would be saved in one day if the entire United States turned off the faucet while brushing.
(c) Put your answer to the previous question in context (compare it to the volume of water in a lake or a swimming pool, for example).
(d) Realistically, you need some water to brush your teeth because you need to get the toothbrush wet and you need to rinse the brush and your teeth. Estimate how much water that involves, per brushing, then redo the estimate in part (b).

Exercise 1.8.18. [W][S][Section 1.3][Goal 1.1][Goal 1.2][Goal 1.5] Lady Liberty.

On May 9, 2009 *The Boston Globe* reported that the Statue of Liberty's crown will reopen:

> Safety and security issues have been addressed, and 50,000 people, 10 at a time, will get to visit the 265-foot-high crown in the next two years before it is closed again for renovation, Interior Secretary Ken Salazar said yesterday. [R22]

Estimate how long each visitor will have in the crown to enjoy the view.

Exercise 1.8.19. [S][Section 1.3][Goal 1.2][Goal 1.5] Look ma! No zipper!

The bag of Lundberg Zipper Free California White Basmati Rice advertises

> We've removed the re-closable zipper from our two pound bags, which will save about 15% of the material used to make the bag, which will save 35,000 lbs. of plastic from landfills every year.

(a) How much plastic is still ending up in landfills?
(b) What fact would you need to figure out how many bags of rice Lundberg sells each year? Estimate that number, and then estimate the answer.

Exercise 1.8.20. [S][Section 1.3][Goal 1.4][Goal 1.5][Goal 1.8] Leisure in Peru.

On page 32 in *The New Yorker* on December 7, 2009, Lauren Collins wrote that late arrivals in Peru are said to amount to three billion hours each year. [R23]

We suspect that the source of Collins's assertion is an article in the July 1, 2007 edition of *Psychology Today* that commented on the campaign for punctuality and included a feature called "Tardiness by the Numbers" that provided the data:

- 107 hours: annual tardiness per Peruvian
- $5 billion: cost to the country
- 84%: Peruvians who think their compatriots are punctual only "sometimes" or "never"
- 15%: think tardiness is a local custom that doesn't need fixing [R24]

(a) According to Collins, how late are Peruvians, in hours per person per day?
(b) Is your answer to the previous question consistent with the numbers in the *Psychology Today* article?
(c) Is the $5 billion "cost to the country" a reasonable estimate?

[See the back of the book for a hint.]

Exercise 1.8.21. [R][S][Section 1.3][Goal 1.1] The white cliffs of Dover.

In his essay "Season on the Chalk" in the March 12, 2007 issue of *The New Yorker* John McPhee wrote

> The chalk accumulated at the rate of about one millimetre in a century, and the thickness got past three hundred metres in some thirty-five million years. [R25]

Check McPhee's arithmetic.

Exercise 1.8.22. [S][C][Section 1.3][Goal 1.1][Goal 1.4][Goal 1.5] Social media and internet statistics.

In January 2009 Adam Singer blogged

> I thought it might be fun to take a step back and look at some interesting/amazing social media, Web 2.0, crowdsourcing and internet statistics. I tried to find stats that are the most up-to-date as possible at the time of publishing this post. [R26]

(a) Read that blog entry, choose a few numbers you find interesting, and make sense of them. Are they reasonable? Are they consistent?

(b) Estimate (or research) what those numbers might be now (when you are answering this question.)
(c) We saw this information on the blog: in March 2008, there were 70 million videos on YouTube. It would take 412.3 years to view all of that YouTube content. Thirteen hours of video are uploaded to YouTube every minute. Can you make sense of these numbers? Are they reasonable? Are they consistent?
(d) Can you locate the source of the statistics above, or other sources that confirm them?

Exercise 1.8.23. [S][Section 1.3][Goal 1.1][Goal 1.3][Goal 1.8] Bottle deposits.

A headline in *The Boston Globe* on July 15, 2010 read "State panel OKs expansion of nickel deposit to bottled water." At the time, Massachusetts required a 5 cent bottle deposit for all bottles containing carbonated liquids. There was a debate about extending the deposit law to other liquids, including bottled water. In the article you could read that

> The Patrick administration, which supports the bottle bill, has estimated the state would raise about $58 million by allowing the redemption of an additional 1.5 billion containers a year, or about $20 million more than the state earns from the current law, and that municipalities would save as much as $7 million in disposal costs. [R27]

(a) Is it reasonable to estimate that 1.5 billion water bottles would be recycled in a year if users paid a nickel deposit on each?
(b) Is $7 million a reasonable estimate of the cost of disposing of 1.5 billion bottles (probably in a landfill) rather than recycling them?
(c) With the data you can estimate the number of water bottles potentially redeemed relative to the number of bottles and cans currently being redeemed. Does the result of the comparison seem reasonable?
(d) The article says the administration estimates that the state will collect $58 million by keeping the deposits paid by the people who don't return the bottles. Use that information to estimate the percentage of bottles that they expect will be recycled.

Note: This bill was defeated in the state legislature.

Exercise 1.8.24. [U][C][Section 1.3] [Goal 1.1][Goal 1.2][Goal 1.8] Drivers curb habits as cost of gas soars.

In *The Boston Globe* on April 21, 2011 you could read that

> [F]amilies are quickly adapting [to increasing gas prices] by carpooling, combining errands to save trips, and curtailing weekend outings, according to organizations that track gasoline consumption. Still, the US Energy Department projects that the average US household will pay $825 more for gas this year than in 2010.
>
> NPD Group Inc., a market research firm, estimates that consumers bought roughly 128 million fewer gallons of gasoline in March than a year earlier. [R28]

Combine reasonable estimates for the increase in gasoline prices, the number of miles driven annually and the average fuel economy of cars to decide whether the $825 figure in the quotation makes sense.

Exercise 1.8.25. [S][Section 1.3][Goal 1.1][Goal 1.2][Goal 1.4][Goal 1.5] The Homemade Cafe.

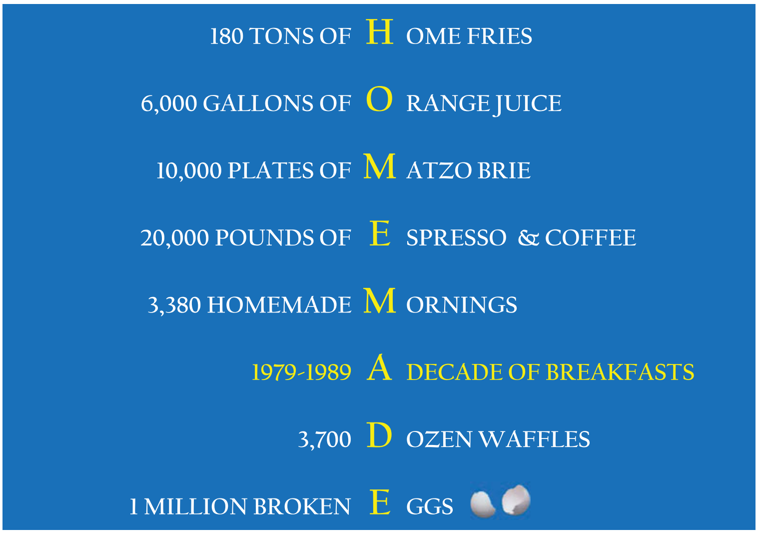

Figure 1.3. The Homemade Cafe [R29]

Figure 1.3 appeared on the back of Berkeley California's Homemade Cafe tenth anniversary tee shirt in 1989.

(a) Check that the numbers there make sense.
(b) Assume that the Homemade Cafe is still in business when you are working on this exercise. What numbers would go on this year's tee shirt?

Exercise 1.8.26. [S][Section 1.3][Goal 1.1][Goal 1.2][Goal 1.3][Goal 1.5] So Many Books, So Little Time.

On Sunday, March 4, 2012 Anthony Doerr asked in *The Boston Globe*:

> Have you ever done the math? If you're lucky enough to have 70 years of literate adulthood, and if you read one book every week, you're still only going to get to 3,640 books. Then you die.
>
> If you consider that the Harvard University Library system's collection is counted in the tens of millions, or that a new book of fiction is published every 30 minutes, 3,640 doesn't seem like so many. [R30]

(a) Confirm that 70 years of reading one book per week would amount to 3,640 books read in a lifetime.
(b) How many people reading one book per week during their lifetime would it take to read all the books in the Harvard University Library system?
(c) If you read one book of fiction each week this year, what percent of all the fiction published this year will you have read?

Exercise 1.8.27. [S][Section 1.3][Goal 1.1][Goal 1.2] Counting car crashes.

The National Safety Council estimates that “21 percent of [automobile] crashes or 1.2 million crashes in 2013 involve talking on handheld and hands-free cell phones.” [R31]

(a) Use the data in the quote to estimate the total number of crashes in the U.S. in 2013.
(b) Check your answer with a web search.
(c) If crashes were evenly distributed across the population, how many would you expect in your community? Does your answer seem reasonable?

Exercise 1.8.28. [R][S][Goal 1.3][Section 1.4] Do parentheses matter?

What does the Google calculator tell you if you accidentally leave out the parentheses in the computation $12/(2 * 3)$?

Exercise 1.8.29. [U][R][Section 1.4][Goal 1.4] Should the U.S. Really Try to Host Another World Cup?

> The proposed budget for the 2010 [Soccer World Cup] games was about $225 million for stadiums and $421 million overall. Expenses have far exceeded those numbers. Reported stadium expenses jumped from the planned level of $225 million to $2.13 billion, and overall expenses jumped similarly from $421 million to over $5 billion. [R32]

How many orders of magnitude off were these estimates? That is, how many places were the decimal points away from where they should have been?

Exercise 1.8.30. [U][Section 1.5][Goal 1.5][Goal 1.1] Check our arithmetic, please.

When we found out in Section 1.5 that Baba Brinkman needed just ten seconds to plant a tree, we decided that his claim was possible.

(a) Verify the statement in the section that “. . . our 100 year estimate should really have been only about two years!”
(b) Verify the next statement: “If it took him a minute per tree rather than 10 seconds he could still have planted a million trees in ten summers.”

Exercise 1.8.31. [S][Goal 1.2][Goal 1.6][Goal 1.8][Section 1.6] The tooth fairy.

How many visits per day does the tooth fairy make in the United States? What’s the daily transaction volume (in dollars) in the tooth fairy sector of the economy?

Exercise 1.8.32. [S][Section 1.6][Goal 1.2][Goal 1.4][Goal 1.8] Paying for college.

In 2006, the ABC program 20/20 told the story of a couple on Los Angeles who put their children through college by collecting and redeeming soda cans and bottles. Their oldest son went to MIT and their two other children attended California state schools. According to the article, the Garcias collected cans and bottles for 21 years with the goal of saving for their children’s college tuition. [R33]

Is this possible?

[See the back of the book for a hint.]

Exercise 1.8.33. [S][Section 1.6][Goal 1.2][Goal 1.4][Goal 1.8] Low flow toilets.

In 1994, a U.S. federal law went in to effect that required all new residential toilets to be “low-flow”, using just 1.6 gallons of water per flush instead of the five gallons per flush of older toilets.

Estimate how much water a household could save in one year by switching to low flow toilets.

Exercise 1.8.34. [S][W][Section 1.6][Goal 1.1][Goal 1.5] Vet bills add up.

The inside back cover of the September/October 2008 issue of *BARk* magazine carried an ad for pet insurance asserting that every ten seconds a pet owner faced a $1000 vet bill.

Is this claim reasonable?

Exercise 1.8.35. [U][Section 1.6][Goal 1.2][Goal 1.6][Goal 1.8] Americans love animals.

In the November 9, 2009 issue of *The New Yorker* Elizabeth Kolbert wrote in a review of Jonathan Safran Foer's *Eating Animals* that there were 46 million dog-owning households, 38 million with cats and 13 million aquariums with more than 170 million fish.

> Collectively, these creatures cost Americans some forty billion dollars annually. (Seventeen billion goes to food and another twelve billion to veterinary bills.) [R34]

Is the twelve billion dollar figure she quotes for veterinary bills consistent with the numbers in the previous problem?

Exercise 1.8.36. [U][Section 1.6][Goal 1.2][Goal 1.4][Goal 1.5][Goal 1.8] Total carbon footprint.

Use the estimates for the seven tasks discussed in Section 1.6 to rank those tasks in order of *total* daily carbon footprint.

Exercise 1.8.37. [S][Section 1.7][Goal 1.1][Goal 1.5][Goal 1.6] How may internet ads?

An employee from Akamai claimed that

> There are 4.5×10^{12} internet advertisements annually. That's two thousand ads per person per year.

(a) Are the figures for the total number of ads and the number per person consistent?
(b) Do you think two thousand ads per person per year is a good estimate?

Exercise 1.8.38. [R][S][Section 1.7][Goal 1.7][Goal 1.8] Metric ton.

A *metric ton*, also known as a *tonne*, is 1000 kilograms.

(a) Is a metric ton a megagram or a gigagram?
(b) How many grams are there in a kilotonne?
(c) How many grams are there in a megatonne?

Exercise 1.8.39. [U][Section 1.7] [Goal 1.4][Goal 1.7][Goal 1.8] e-reading.

In December, 2014 Amazon offered a Kindle e-reader with

> **Storage:** 16GB (10.9GB available to user) or 32GB (25.1GB available to user), or 64 GB (53.7GB available to user)

(a) Compare the percentage of storage available to the user for each of these options.
(b) Estimate the number of e-books you could store on the 64 GB Kindle.
(c) Estimate the size of *Common Sense Mathematics* in MB.

Exercise 1.8.40. [U][Section 1.7][Goal 1.4][Goal 1.7][Goal 1.8] Personal storage.

(a) How many bytes of storage are there on the hard drive of your computer (or tablet or smartphone, or some device you use regularly)? Is that best measured in megabytes or gigabytes?
(b) If you have a thumb drive or flash memory stick, what's its capacity?

Exercise 1.8.41. [U][Section 1.7][Goal 1.4][Goal 1.7][Goal 1.8] Backing up the Library of Congress.

How many 200 gigabyte computer memories would you need to store the books in the Library of Congress?

Exercise 1.8.42. [U][Section 1.7][Goal 1.6][Goal 1.7] giga-usa.

The website `www.giga-usa.com/` advertises itself as an

> Extensive collection of 100,000+ ancient and modern quotations, aphorisms, maxims, proverbs, sayings, truisms, mottoes, book excerpts, poems and the like browsable by 6,000+ authors or 3,500+ cross-referenced topics. Extensive collection of 100,000+ ancient and modern quotations, aphorisms, maxims, proverbs, sayings, truisms, mottoes, book excerpts, poems and the like browsable by 6,000+ authors or 3,500+ cross-referenced topics. [R35]

Is the web site properly named?

Exercise 1.8.43. [R][S][Section 1.7][Goal 1.4][Goal 1.5][Goal 1.7][Goal 1.8] Data glut.

In the article from *The Boston Globe* on February 24, 2003 with the long headline

> Data glut as gene research yields information counted in terabytes. Researchers struggle to visualize and process it while technology businesses scramble to profit from it.

you could read that

> [Peter Sorger's] bioengineering lab produces a terabyte of data in a typical month. [R36]

(a) At what rate in bytes per minute is the lab producing data? Write your answer with the appropriate metric prefix and the appropriate level of precision.
(b) If the lab has been producing data from the time the article appeared to the present, how much has accumulated now?
(c) When will a petabyte of data have accumulated? Do you believe your prediction?
(d) When will an exabyte of data have accumulated?

Exercise 1.8.44. [S][C][Section 1.7][Goal 1.7][Goal 1.6] Zettabytes.

On September 7, 2010 an editorial in *The Boston Globe* said that

> The total amount of digital storage worldwide is approaching 1 zettabyte, or 1 million times the contents of the Earth's largest library. [R37]

A zettabyte is 1,000,000,000,000,000,000,000 bytes. The editor provides the comparison to help her reader make sense of that nearly incomprehensible number.

Check her arithmetic. Are the numbers consistent—that is, do they agree with each other when you compare them? Do they make sense?

Exercise 1.8.45. [U][C][Goal 1.6][Section 1.7] [Goal 1.7][Goal 1.8] Zettabytes redux.

On August 6, 2011 Kari Kraus wrote in *The New York Times* that

> We generate over 1.8 zettabytes of digital information a year. By some estimates, that's nearly 30 million times the amount of information contained in all the books ever published. [R38]

Are the two estimates in this quotation (1.8 million zettabytes, 30 million times …) consistent with each other?

Exercise 1.8.46. [W][S][Section 1.1][Goal 1.1] Waiting for the light to change.

In the Pooch Cafe comic strip on August 27, 2012 Poncho the dog is sitting in the car with his master Chazz. He says "Did you know the average person spends six months of their life waiting at red lights?" (You can see the strip at `www.gocomics.com/poochcafe/2012/08/27`)

What do you think of Poncho's estimation skills?

Exercise 1.8.47. [U][Section 1.1][Goal 1.1][Goal 1.2] Killer cats.

In the article "The impact of free-ranging domestic cats on wildlife of the United States" in January 2013 in *Nature Communications* Scott R. Loss, Tom Will and Peter P. Marra offered an

> … estimate that free-ranging domestic cats kill 1.4–3.7 billion birds and 6.9–20.7 billion mammals annually. Un-owned cats, as opposed to owned pets, cause the majority of this mortality. [R39]

Make sense of those numbers. Consider kills per cat or kills per day, or kills per day in your community.

Exercise 1.8.48. [S] Viagra, anyone?

A 2014 Viagra ad on TV stated that more than 20 million men already use Viagra. Use the U.S. population pyramid in Figure 1.4 to argue whether or not this claim seems reasonable. Be explicit about any assumptions you make about the age groups of men who might typically use this drug.

Exercise 1.8.49. [U] Lots of olives?

On August 23, 2014 the Associated Press reported that

> In the 1980s, American Airlines chief executive Robert Crandall famously decided to remove a single olive from every salad. The thought was: passengers would not notice and American would save $40,000 a year. [R41]

(a) About how many olives could Crandall buy for $40,000?
(b) Is your answer to the previous question in the same ballpark as the number of American Airlines passengers in 1980?

Exercise 1.8.50. [U][C][Section 1.3][Goal 1.1][Goal 1.2][Goal 1.5][Goal 1.8] Nuclear bombs.

"Kiloton" and "megaton" are terms you commonly hear when nuclear bombs are being discussed. In that context the "ton" refers not to 2,000 pounds, but to the explosive yield of a ton of TNT.

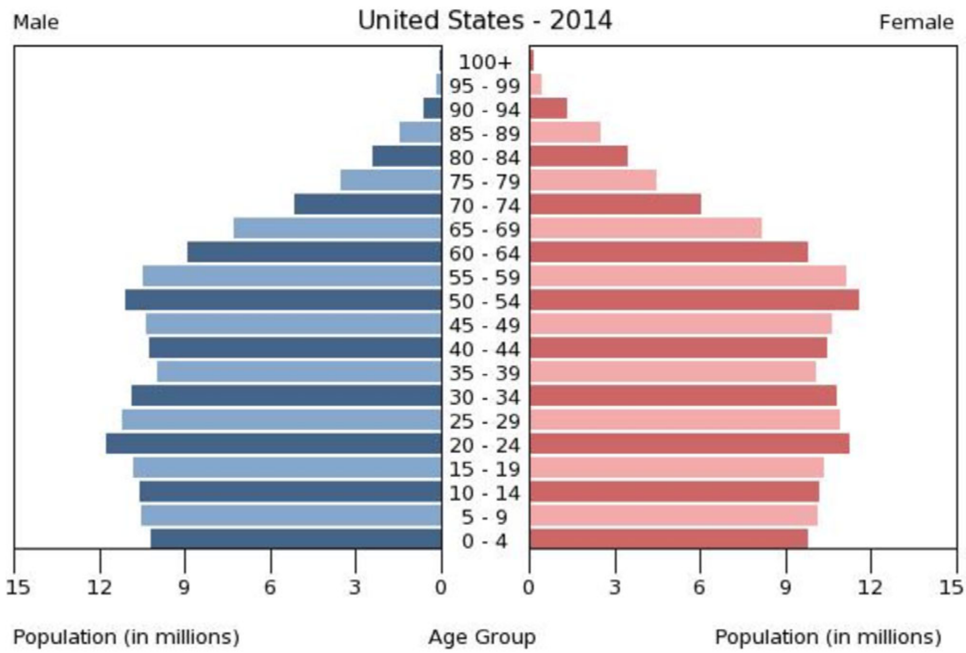

Figure 1.4. U.S. Population pyramid [R40]

(a) What was the explosive yield of the only (two) atomic bombs ever used in war?
(b) How does the explosive yield of the hydrogen bombs in the current arsenals of the United States and Russia (and other countries) compare to that of those first atomic bombs?
(c) Estimate the destructive power of the world's current stockpile of nuclear weapons, in terms easier to grasp than kilotons or megatons or gigatons or ….

Review exercises

Exercise 1.8.51. [A] Do each of these calculations by counting zeroes. Use the Google or Bing calculator (or another calculator) to check your answers.

(a) One million times one billion.
(b) Four hundred time three thousand.
(c) Two billion divided by two hundred.
(d) One-tenth times five thousand.
(e) Four thousand divided by two hundred.
(f) Twelve thousand times two hundred.
(g) $10{,}000 \times \frac{2}{1000}$.
(h) $450{,}000 \times 100$.
(i) $\frac{50{,}000}{200}$.
(j) $\frac{4{,}000{,}000{,}000{,}000}{2{,}000{,}000}$.

Exercise 1.8.52. [A] Use rounding to estimate each of these quantities. Then check your answers.

(a) The number of feet in ten miles.
(b) The number of minutes in a week.
(c) The number of inches in a mile.
(d) The number of yards in a mile.
(e) The number of seconds in a month.

Exercise 1.8.53. [A] Answer each of these questions without using pencil and paper (or a calculator).

(a) There are twelve cans of soda in a case. If I buy five cases of soda, how many cans will I have in total?
(b) A case of bottled water contains 24 bottles. About how many bottles are there in four cases?
(c) At the market my cart contains: three bags of cereal at $2.19 each; a gallon of milk for $2.99; a pound of potatoes that costs $3.50 and a $3.99 bag of apples. Will my total be more than twenty dollars?
(d) At our local deli, a bagel costs $1.19. How much would five bagels cost? If the deli offers six bagels for $6.99, is that a better deal?
(e) It costs $2.50 to ride the New York City subway. A seven-day unlimited pass costs $29. How many rides in a seven day period should you take before it's a better deal to buy the pass?
(f) I have a $100 gift card for a department store. Can I buy a pair of shoes for $19.99, two shirts that are $12.59 each and three pairs of pants that each cost $20.50?

Exercise 1.8.54. [A] Convert

(a) One kilometer into meters.
(b) One megameter into kilometers.
(c) One terabyte into megabytes.
(d) Three megawatts into kilowatts.
(e) Five gigabytes into kilobytes.
(f) One thousand kilograms into megagrams.